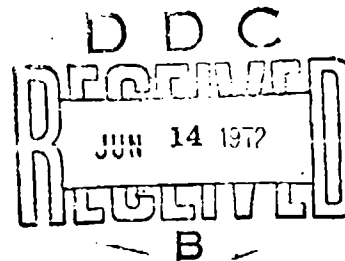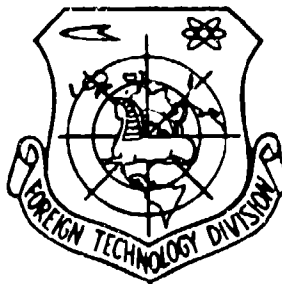AD743142

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MEASUREMENTS IN PLASMA DIAGNOSTICS USING
A MACH ZEHNDER INTERFEROMETER

by

V. L. Exner

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13. ABSTRACT

In this survey the principle of the optical interference method
is described and formulas for determining the value of the
interference fringes shift are derived. Dispersion equations
stating the relation between the refractive index N and the gas
density P, the given free electrons concentration , and the
product of the concentration of excited atoms in the state I
and the respective strength of the given oscillator are pre-
sented. A Mach-Zehnder interferometer is the most frequently
used apparatus for the interference measurements of these
quantities. The interferometer with a crossed spectrograph
using white light according to Rozhdyestvenskiy is described.
Less known methods to obtain and evaluate the interferograms
are discussed. A considerable part of the paper concerns the
concrete applications of optical interferometry in general
and the Mach-Zehnder interferometer in particular in plasma
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MEASUREMENTS IN PLASMA DIAGNOSTICS USING A MACH-ZEHNDER INTERFEROMETER

Summary: In this review article the principle is described of the classical optical interference method, and formulas are derived for determining the shift of the interference fringes. Dispersion formulas are presented that express the relationship between the refractive index n for a light beam of wavelength λ, and the characteristics of the medium through which the beam passes: the density ρ, the free electron concentration N_e, and the product $N_i f_{ik}$ of the concentration of excited atoms in state i and of the corresponding oscillator force. The Mach-Zehnder interferometer is the apparatus most frequently used for interference measurements of these characteristics. Rozhdestvenskiy's configuration of an interferometer with a crossed spectrograph, using white light, is described. Less known methods of obtaining and evaluating interferograms are discussed. Considerable space is devoted to the specific applications of optical interferometry and of the Mach-Zehnder interferometer in plasma diagnostics.

CONTENT

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1. Introduction

When measuring the characteristics of partially or completely ion-
ized gases, it is particularly important to select methods that least
disturb the space of the measured region (for example, with metal elec-
trodies, holders, and leads) and result only in a small exchange of
energy during measurement. Optical methods meet these requirements
for many of the measurements that occur in plasma physics. The purpose
of this article is to acquaint the reader in greater detail with one
of these methods, the classical interference method, and particularly
with the possibilities that the Mach-Zehnder interferometer offers.

In recent years, there has been an increase in the number of
works -- particularly in the study of shock waves, high-frequency dis-
charges, and hot plasmas -- that employ specifically this apparatus to
measure the changes in density, temperature, and electron concentra-
tion, respectively in the concentration of excited atoms. The import-
ance of the otpical interference method has been enhanced by the dis-
covery of lasers and of holography that considerably expand the problems
to which optical methods can be applied. Combinations of these differ-
ent optical methods and apparatuses are opening new ways for conducting
very demanding experiments and quantitative measurements.

This article explains the principle of the interference method
(Section 2), presents the relationship between the refractive index and
the characteristics of the medium (Section 3), and describes the Mach-
Zehnder interferometer (Section 4). In subsequent sections (5--8) we

describe the methods of obtaining and evaluating interferograms, particularly the methods that are less known in Czechoslovakia. Section 10 is devoted to the applications of the interference method in plasma diagnostics.

2. Principle of the Interference Method

The refractive index of light passing through a medium depends on the concentration of oscillating states capable of reacting with the photons, and on the passing light itself. By accurately measuring the refractive index or its change, it is possible to determine the changes in, or the absolute values of, the medium's characteristics. Three types of optical methods [1] are based on this principle: the shadow-graph technique, the schlieren method (from the German word Schliere = = optical inhomogeneity), and the interference method.

Assume that a beam of light is passing through a medium with a changing refractive index n. The passage of the beam of light will be governed by Fermat's principle

$$\delta \int n(x, y, z)\, ds = 0 .$$

The solution of this equation gives Euler's formulas

$$\frac{d}{ds}\left(n\frac{dx}{ds}\right) = \frac{\partial n}{\partial x}, \quad \frac{d}{ds}\left(n\frac{dy}{ds}\right) = \frac{\partial n}{\partial y}, \quad \frac{d}{ds}\left(n\frac{dz}{ds}\right) = \frac{\partial n}{\partial z} \qquad (2.1)$$

and Snell's law of refraction. The effect of the optically inhomogeneous region upon the passing light wave will be such that on a screen located beyond the region it is possible to record either the variations of the wave angle or the shift of the beam's point of incidence, or -- in suitably arranged optical systems -- the phase delays.

In optical systems in which variations occur in the screen's illumination as a function of the phase delay, it is necessary to employ interference phenomena because only they enable us to record the phase delay of coherent light waves.

A characteristic property of interferometers is that the investigated region in an inhomogeneous distribution of the refractive index is located so that only one of two coherent light beams passes through it. The other beam remains unchanged and serves for comparison.

Assume that on the screen we have image Q of some point P in the investigated region. In the idle condition, when there is no variation of the refractive index over the given region, image Q lies on interference fringe α, for which the following relationship applies in the case of monochromatic lights of wavelength λ_0:

$$\alpha \lambda_0 = n_0\, \Delta s ,$$

- 3 -

where n_0 is the refractive index, and Δs is the difference between the path of the beams in the two arms of the interference system. In a system in which the refractive index changes, image Q' of point P lies on fringe β, for which the following relationship applies::

$$\beta \lambda_0 = \alpha \lambda_0 + c_0 \tau ;$$

here c_0 is the velocity of light near the investigated region without optical inhomogeneity, and time τ characterizes t phase delay. The difference $S = \beta - \alpha$ is called the fringe shift and is a function of the position of image Q' on the screen or photographic plate, and hence a function of the position of point P, under the frequently employed assumption that refraction inside the optically inhomogeneous region causes the beam to change its path only slightly. The fringe shift is then proportional to the phase delay:

$$S(x, y) = \frac{c_0}{\lambda_0} \tau = \frac{c}{\lambda} \tau ,$$

c and λ apply in vacuo. The evaluation of interferograms is based on the dependence of τ upon the refractive index $n(x, y, z)$, and thus on the dependence of fringe shift $S(x, y)$ upon the refractive index $n(x, y, z)$.

Furthermore, assume a rectangular system of coordinates x, y, z, where the z axis corresponds to the direction along which the incident monochromatic light enters the observed optically inhomogeneous region. Let the incoming plane $z = B_1(x,y)$, and the exit plane $z = B_e(x, y)$. Then the system of Euler's formulas (2.1) can be written in the form of two differential equations that define x and y as functions of z:

$$x'' = (1 + x'^2 + y'^2)(l_x - x'l_z), \qquad (2.2)$$

$$y'' = (1 + x'^2 + y'^2)(l_y - y'l_z) ; \qquad (2.3)$$

the primes designate derivatives according to z, and l_x, l_y and l_z are are the components of grad $n(x, y, z)$. If

$$x = \varphi(\xi, \eta; z) \quad \text{and} \quad y = \psi(\xi, \eta; z) ,$$

then the solutions of Equations (2.2) and (2.3) at the point where the beam of light enters the system are $x = \xi$, $y = \eta$, $z = \zeta = B_1(\xi, \eta)$, $x' = y' = 0$, and if $z = \zeta_1$ is the root of the equation

$$z = B_e[\varphi(\xi, \eta; z), \psi(\xi, \eta; z)] ,$$

then the parameters of the beam at the point where it enters the optical inhomoageneity are:

- 4 -

$$x = \zeta_1 = \varphi(\xi, \eta; \zeta_1), \; y = \eta_1 = \psi(\xi, \eta; \zeta_1),$$

$$x' = p_1 = \varphi'(\xi, \eta; \zeta_1), \; y' = q_1 = \psi(\xi, \eta; \zeta_1).$$

Then the phase delay will be

$$\tau = \frac{1}{c} \int_\zeta^{\zeta_1} \{n[\varphi(z), \psi(z), z] \sqrt{(1 + [\varphi'(z)]^2 + [\eta'(z)]^2)} - n_0\} \, dz . \qquad (2.4)$$

If we assume that the changes in the beam's direction are infinitely small, then Equation (2.4) can be simplified:

$$\tau \approx \frac{1}{c} \int_\zeta^{\zeta_1} [n(x, y, z) - n_0] \, dz , \qquad (2.5)$$

because then

$$\varphi'(z) = \psi'(z) = 0, \; \varphi(\xi, \eta; z) = \xi, \psi(\xi, \eta; z) = \eta .$$

Equations (2.4) and (2.5) do not include the stepwise changes in the refractive index that follow Snell's law of refraction.

In the evaluation of interferograms, the fringe shifts S(x, y) are measured:

$$S(x, y) = \frac{1}{\lambda} \int_\zeta^{\zeta_1} [n(x, y, z) - n_0] \, dz . \qquad (2.6)$$

Here the only unknown is the change of the refractive index, and we start out from this basic equation to evaluate the photographs taken of the interference fringes.

3. Dispersion Formulas

Dispersion formulas express the relationship between the refractive index n for light of wavelength λ, and the characteristics of the medium. In the following we will present the basic formulas that are used in interference measurements.

The oscillator's equation of motion is:

$$m\xi'' + h\xi' + f_0\xi = e(X + 4\pi\alpha eN\xi) ,$$

where ξ is the component of the oscillator's displacement, m is the oscillator mass, $-f_0\xi$ is the quasielastic force, e is the charge, X is the electrical field, N is the number of oscillators per cubic centimeter, h denotes the damping of the oscillator, and $\alpha = 0$ or $1/3$. P. Drude, H. Lorentz and M. Planck derived from this equation of motion the final

results of the theory of electromagnetic wave dispersion that subsequently were confirmed also by quantum mechanics:

$$n^2 - k_0^2 = 1 + \sum \frac{a(\lambda^2 - \lambda'^2)\,\lambda^2}{(\lambda^2 - \lambda'^2)^2 + b_0\lambda^2},$$

$$2nk_0 = \sum \frac{ab_0\lambda^3}{(\lambda^2 - \lambda'^2)^2 + b_0\lambda^2}.$$

The summation is over all oscillating states.

In these equations n is the refractive index, and k_0 is the absorption coefficient defined by the equation

$$I = I_0 \exp\left(-\frac{4\pi k_0}{\lambda} d\right),$$

where I_0 is the intensity of light incident upon the body, and I is the intensity of light after passage through a layer of thickness d. Into the equations Drude substitutes

$$\lambda'^2 = \lambda_v^2,\quad a = \frac{Ne\lambda_v^2}{\pi mc^2},\quad b_0 = \frac{h}{m}\frac{\lambda_v^2}{2\pi c}$$

and Lorentz and Planck subsitute

$$\lambda'^2 = \lambda_r^2(1 + \alpha a),\quad a = \frac{Ne\lambda'^2}{\pi mc^2},\quad b_0 = \frac{h}{m}\frac{\lambda'^2}{2\pi c};$$

where λ_v is the wavelength corresponding to the natural frequency of the independent oscillator

$$\lambda_v^2 = 4\pi^2 c^2 \frac{m}{f_0}.$$

Light absorption in the measured region usually can be neglected. Sellmeyer [2] simplified the dispersion formula on the basis of the oscillator equation of motion:

$$m\zeta'' = f_0\zeta = eX.$$

The dispersion formula then assumes the form

$$n - 1 = \tfrac{1}{2}\sum \frac{a\lambda^2}{\lambda^2 - \lambda'^2}$$

The summation is over all oscillator states. Rozhdestvenskiy verified this formula in practice, on a doublet of sodium, with an accuracy better than 2.5 percent [3] (only the decisive contributions are taken into account very close to the spectral lines).

From Sellmeyer's dispersion formula, equations are derived for the dependence of the refractive index upon the characteristics of the medium: its density ρ, electron concentration N_e, and the product of the concentration of excited states or ions N_i, and of the corresponding oscillator force f_{ik}. The individual equations are written as follows:

For the density of the gas or vapor

$$n - 1 = A\rho,\tag{3.1}$$

where A is a constant for the given configuration and the given gas or vapor, over a wide range of densities, and it depends on the wavelength of the light;

For the concentration of free electrons

$$n - 1 = \frac{N_e e^2 \lambda^2}{2\pi m c^2}\tag{3.2}$$

And for the product $N_i f_{ik}$

$$n - 1 = \sum_{i,k} \frac{e^2 N_i f_{ik}}{2\pi m c^2} \frac{\lambda_{ik}^2 \lambda^2}{\lambda^2 - \lambda_{ik}^2},\tag{3.3}$$

where m is the electron mass, and λ_{ik} is the wavelength characteristic of the transition between states i and k. The quantities f_{ik} are related to Einstein coefficients A_{ik} and to the probabilities of spontaneous transition from excited k to lower state i, according to the following formula [4]:

$$f_{ik} = \frac{3mc\lambda_{ik}}{8\pi^2 e^2} A_{ik}.$$

The density ρ is related by the equations of state with other quantities (temperature, pressure, etc.), and these too can be measured with the help of the changes in the refractive index, according to Equation (3.1).

4. Interferometer

The Mach-Zehnder interferometer is the most suitable for the considered interference measurements of the refractive index. In the Soviet literature this apparatus is often referred to as the Rozhdestvenskiy interferometer (Fig. 1) [3, 5--7]. This instrument was developed at the end of the 19th century, from Jamin's interferometer. The Mach-Zehnder interferometer retains all the advantages of the Jamin interferometer. In addition, it is much more stable over time, because intensive light passes only through thin plates that quickly heat up and then no longer change their temperature. The interferometer gives fixed fringes for hours at a time. Another advantage of the interferometer is the ease of changing the length of the two interfering beams.
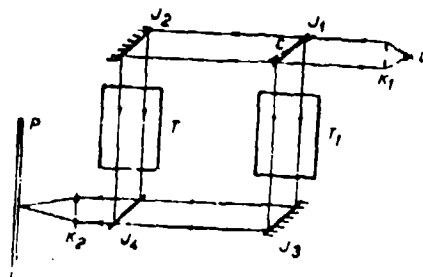
Figure 1.   The Mach-Zehnder interferometer.

· ·Although in principle the refractive index can be measured also
with the Michelson interferometer, it is not used because of the unsuit-
able configuration of its measuring cell and its compensating cell.

The Mach-Zehnder interferometer consists of four glass plates J_1
through J_4 that are mutually parallel. Their centers lie in a plane,
in the apices of a rectangle. J_2 and J_3 are mirrors, while J_1 and J_4
are semitransparent plates.

One arm of the interferometer, between J_2 and J_4, is arranged so
that the beam of light passes through the measured region T that is
optically inhomogeneous. When necessary, a compensating cell T_1 is
interposed in the other arm, between J_1 and J_4. Interference fringes
appear on screen P. If one of the semitransparent plates is rotated a
small angle ϵ, and if the basic fringe in the interferometer without an
optical inhomogeneity in the measuring arm is parallel with the axis of
rotation of semitransparent plate J_1 or J_4.

The optical system of the interferometer proper consists of a
source of light that shines either constantly (for a longer period of
time) or for only a very short time, in accordance with the nature of
the interferometer's application (for example, only a few microseconds
in certain experiments with shock waves). Furthermore, the optical sys-
tem includes light filters, and a condenser that forms a parallel beam
of rays which enter the interferometer usually at an angle of 45°. The
field is proportional to the cosine of this angle, and hence even smaller
angles are desirables in some instances [1]. The outgoing parallel beam
of rays is displayed by a lens on the screen. If the interference pat-
tern is photographed, this lens can be directly the camera objective.

Either monochromatic or white light is used for measurements with
the interferometer. Measurements in monochromatic light are less de-
manding with respect to the interferometer's optical properties. In ad-
dition, monochromatic light gives far more fringes. White light, on the
other hand, has very strong central fringes that are white in color,
whereas the other fringes are rainbow-like. The pronounced central
fringes make for much easier orientation in measuring. White light is
employed also in L. Puccianti's method [8].

Adjustment of the interferometer is time-consuming. A number of methods has been developed (see, for example [1, 3, 9]) that considerably shorten and simplify the process. But once the interferometer is adjusted, in further use it is very stable, provided it is suitably designed.

5. Routine Methods of Evaluating Interferograms

If the changes in the refractive index are known, Equations (3.1), (3.2) and (3.3) enable us to determine the changes in the parameters of an optically inhomogeneous medium. The changes in the refractive index are computed from Equation (2.6), at known values of the fringe shift.

In the two-dimensional case, which is by far the more common, it is assumed that the refractive index is a function of only coordinates x, y, and that in the direction of the beam's propagation (axis z) the optical inhomogeneity is constant for every x, y. If we denote by d the thickness of the optical inhomogeneity, we obtain

$$n(x, y) = n_0 + \frac{\lambda}{d} S(x, y).$$ (5.1)

The two-dimensional case of optical inhomogeneity is the most simple. Equation (2.6) has a unique solution for the change in the refractive index also in the case of an axially symmetrical optical inhomogeneity. This case is particularly interesting from the viewpoint of application to plasma physics:

Let axis x, which is perpendicular to the direction of light propagation, be the axis of symmetry. Then $n = n(x, r)$, where $r = \sqrt{(y^2 + z^2)}$. For element dz we obtain:

$$dz = \frac{r\,dr}{\sqrt{(r^2 + y^2)}}$$

and for every $x = \text{const}$ we get an equation of the Abel type that expresses the phase shift

$$S(y) = \frac{1}{\lambda} \int_y^R [n(r) - n_0] \frac{d(r^2)}{\sqrt{(r^2 - y^2)}},$$ (5.2)

where R is the radius of the inhomogeneity. With the help of the operator

$$\int_y^R [\ldots] \frac{d(y^2)}{\sqrt{(y^2 - u^2)}}$$

we rearrange Equations (5.2) as

$$n(r) - n_0 = -\frac{\lambda}{\pi} \int_r^R \frac{dS(y)}{\sqrt{(y^2 - r^2)}}$$

(5.3)

To solve the right side of Equation (5.3), a planimeter was used in Germany [10], but in the United States a numerical method was employed that presupposed the future use of electronic computers [11, 12].

In less simple cases the solution of Equation (2.6) would become rather complicated, and in general the solution would not be unique.

6. Puccianti's Method

For his outstanding measurements of anomalous dispersion in metal vapors, D. S. Rozhdestvenskiy [3, 13] used an interferometer with a crossed diffraction-grating spectograph, as shown in Fig. 2. E. Mach originally proposed this arrangement in 1875. G. von Osnobischin used ·it for qualitative measurements. It was rediscovered independently by L. Puccianti [8], after whom this configuration is the most often called, particularly in the Russian literature. We will briefly describe this method which in its applications permits an increase in the sensitivity of the interference measurements. This method belongs among those where it is necessary to assume two-dimensional optical inhomogeneity; in the direction of the passage of light (along axis z) the refractive index is assumed to be constant for every x, y (the two-dimensional case).

Two coherent beams of white light interfere and form horizontal interference fringes on the vertical slit of the spectroscope (Fig. 2). In the spectroscope's ocular it is then possible to see a spectrum with horizontal fringes. The central white fringe, corresponding to the difference between the optical paths of the coherent beams, is strictly horizontal. But the higher-order fringes are inclined, because the differences between the optical paths are proportional to the refractive index that depends on the wavelength; for red light they are greater than for violet light. Let us consider axis y that is perpendicular and parallel to the spectroscope's slit. The optical path difference Δs changes along the slit. Let $y = 0$ for the zero fringe, and let us assume that Δs is proportional to y:

$$\Delta s = by.$$

For the first fringe, $\Delta s = \lambda$; for the second fringe, $\Delta s = 2\lambda$, etc. Then

$$by_k = \lambda k,$$

(6.1)

if we denote the coordinate of the k-th fringe as y_k. If now the value of the refractive index in one arm of the interferometer changes from value n_0 (which is the same as in the compensator located in the interferometer's other arm) to a value $n(\lambda)$ over length d, then Equation (6.1) will assume the following form:
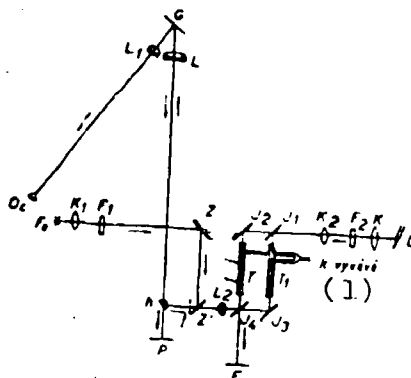
Figure 2. Rozhdestvenskiy's configuration of an interferometer
            and a crossed spectrograph [3, 13], used for measure-
            ments according to Puccianti's method. White light
            (L) forms interference fringes in slit f, propagates
            through reflecting prism h to objective L and grating
            C, and the light reflected back passes under prism h
            and forms a spectrum on photographic plate P. F_2 is
            a thick filter containing water and $K_3Cr_2O_7$; it ab-
            sorbs the thermal rays and the violet part of the
            spectrum. T is the measuring area (cell), T_1 is the
            compensator. Of the interferometer's outgoing two
            pairs of interfering beams it is expedient to use
            the second pair that in the direction J_2J_4 forms, by
            shadow projection, a large interference pattern on
            the screen. Arc F_c, with oblique electrodes, forms
            a calibration spectrum over the interference spectrum.
            The calibration spectrum is controlled in ocular O_c.

Key:

(1) To vacuum pump

$$b y_k = k\lambda + [n(\lambda) - n_0]\, d \; ;$$

for the zero fringe ($k = 0$) we get

$$y_0 = \frac{d}{b}\,[n(\lambda) - n_0] \,,$$

which is directly the dispersion formula. The zero fringe will then
represent in the ocular the course of the dispersion formula; the
scale will depend on quantities d and b, and n_0 is usually 1.

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7. Rozhdestvenskiy's Method of Hooks

With this method it is much easier to measure the product $N_i f_{ik}$. We will briefly explain its essence [3]. When measuring anomalous dispersion by means of an interferometer with a crossed spectrograph (Fig. 2), in the spectrograph's ocular we see the interference fringes in the proximity of a strong absorption line, as shown schematically in Fig. 3a. The interference fringes always shift upward in the direction from the small wavelengths toward the line (from left to right).
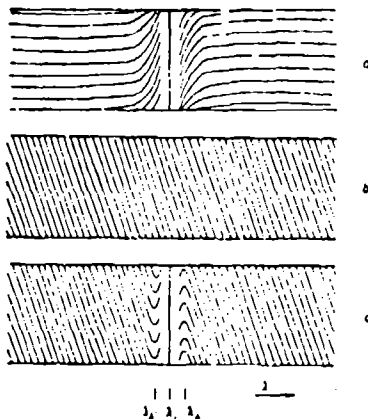
Figure 3. The method of hooks, illustrated.

Let us now assume that in the measuring arm of the interferometer we do not have vapor or gas, but a plane and parallel thin plate. The originally horizontal fringes will now be inclined from above downward and from left to right (Fig. 3b), i.e., in the opposite direction than the inclination of the fringes (curves) of anomalous dispersion. The inclination of the fringes is caused by the fact that the central white fringe has shifted very high. The thicker the plate, the higher the central fringe shifts, and the steeper will be the inclination of the fringes.

When measuring according to the method of hooks, the measuring arm of the interferometer contains the gas or vapor to be measured, and also the thin plane and parallel plate. In the spectrograph's ocular we observe interference fringes that form "hooks" about the absorption line, as shown in Fig. 3c. At the points of the hooks' minima (or maxima), the inclination of the fringes due to the plane and parallel plate is exactly equal to the inclination of the fringes caused by the anomalous dispersion of the gas or vapor. If the inclination caused by the plate is known, then it is possible to compute the inclination of the dispersion curve. For the product Nf (the concentration of states corresponding to line λ, times the oscillator force), Rozhdestvenskiy derived the formula:

$$Nf = \frac{\pi m c^2}{e^2 d \lambda_i^3} K(\lambda_h - \lambda_h') \; ;$$

where $\lambda_h - \lambda_h'$ is the wavelength distance between the maxima and the
minima of the hooks, and K is the method's constant that accounts for
the characteristics of the plane and parallel plate. The evaluation of
interferograms with hooks has been expanded in [14] to permit the use
of a much greater portion of the spectrum, and this enhances the accur-
acy of the measurements.

8. Method of Superposition

This method, also known as "transparency," is used to measure
changes in temperature over hot surfaces. The same photographic plate
is used to photograph the interference fringes over a cold surface, and
then the interference pattern after the surface has been heated. The
so-called "blurs" -- i.e., the geometric loci of points at which the
interference pattern shifts so that the light part of the fringes is
specifically where the dark part had been in the interference pattern
of the cold surface -- are mutually separated by a distance correspond-
ing to the shifting of the fringes by one. The shape of these "blurs"
directly corresponds to the isotherms over the hot surface.

In the same manner it is possible to obtain the direct course of
the isotherms when measuring with the interferometer in the so-called
basic position. In this case all the mirrors are parallel, and the
fringe -- when there is no optical inhomogeneity in the measuring arm
of the interferometer (Fig. 1) -- is endlessly wide, in practice wider
than the linear dimensions of the field of view. An optical inhomoge-
neity -- i.e., the heating of the surface -- causes a phase delay in
the light and forms interference fringes that are again direct isotherms.
It is convenient that in the method of superposition, as well as when
measuring with the interferometer in the basic position, the separation
between two neighboring isotherms corresponds to the same difference in
temperature. This follows from the properties of interference fringes
and from the dependence in Equation (3.1).

In work [15], the method of superposition was used in measurements
with white light, on an interferometer and crossed spectrograph accord-
ing to the configuration in Fig. 2. In measurements with white light,
the method of superposition increases the accuracy by 1.5 orders. We
will explain the essence of this method on the simple case when the den-
sity of the vapor or gas changes by a quantity $\Delta\rho = \rho - \rho_0$.

According to Equation (3.1), the density dependence of the refrac-
tive index is: $n - 1 = A\rho$. If the density changes by $\Delta\rho$, this means a
change in the optical path (in terms of wavelength) by $dA\Delta\rho/\lambda$, where d
is the length of the column of gas or vapor. The two sets of interference
fringes on the photograph form loci where the fringes become more

pronounced, and loci where the fringes become blurred. If these "blurs" are used to evaluate the measured quantities, the useful portion of the spectrum is expanded considerably, and the accuracy is enhanced. For two "blurs"

$$dA\,\Delta\varrho = \frac{2k_1 + 1}{2}\lambda_1\,,$$

$$dA\,\Delta\varrho = \frac{2k_2 + 1}{2}\lambda_2\,,$$

$$k_2 = k_1 + 1\,;$$

where λ_1 and λ_2 are the wavelengths of the blurred loci k_1 and k_2. Wavelengths λ_1 and λ_2, and also 1, which is the number of blurs between k_1 and k_2, are read from the interferogram. From the equations it is then possible to determine unambiguously $\Delta\rho$, k_1 and k_2.

Similar equations can be written also for the more complicated cases of influencing the refractive index, for example, for a simultaneous change in the density of the gas or vapor and in the electron concentration. In this case the method of superposition offers a further advantages: the "blurs" caused by a change in density, and the ones caused by a change in electron concentration can be distinguished very easily, due to the form of the dispersion formulas in Equations (3.1) and (3.2).

Let us mention a very interesting practical case when the change in the optical path is so small that the interference fringes shift by only a fraction of the separation between fringes. In this case, use is made of the feasibility of influencing the interference fringes in the compensator arm of the interferometer. Let us change the density in the compensator by $\Delta\rho_0$ in length d_0, and let us photograph the interference fringes before and after the change, on the same plate, according to the method of superposition. Then the following expression will apply to the k-th "blur" at wavelength λ_{k_0}:

$$\frac{d_0 A_0\,\Delta\varrho_0}{\lambda_{k_0}} = \frac{2k + 1}{2}\,,$$

where A_0 is a constant. From this relationship it is possible to compute λ_{k_0}, without preparing a calibrating interferogram. Here again the actual interferogram is obtained by the method of superposition: on the same plate we photograph the interference fringes before and after a change in the density by $\Delta\rho_0$ in the compensator, and a change in the parameter in the measuring arm. For a change in the density by $\Delta\rho_x$, for example, we get for the k-th blur:

$$\frac{d_0 A_0\,\Delta\varrho_0 \mp dA\,\Delta\varrho_x}{\lambda_k} = \frac{2k + 1}{2}\,. \tag{8.1}$$

The difference $\lambda_{k_0} - \lambda_k$ (the shift of the k-th blur) then serves for the determination of $\Delta\rho_x$:

$$\lambda_{k_0} - \lambda_k = \pm \frac{2dA}{2k+1} \Delta\rho_x. \qquad (8.2)$$

For a change of the electron concentration by N_e we obtain the change in the optical path, in wavelengths,

$$\frac{dN_e e^2}{2\pi m c^2} \lambda,$$

and hence an equation similar to Equations (8.1):

$$\frac{d_0 A_0 \Delta\rho_0}{\lambda_k} - \frac{dN_e e^2}{2\pi m c^2} \lambda_k = \frac{2k+1}{2}.$$

Then the following relationship will apply to the shift of the k-th "blur" due to a change in the electron concentration by N_e:

$$\lambda_{k_0} - \lambda_k = \frac{2}{2k+1} \frac{dN_e e^2}{2\pi m c^2} \lambda_k^2.$$

By this method it is feasible to measure also the simultaneous changes $\Delta\rho_x$ and N_e.

The method of superimposing interference patterns occupying spectral regions close to absorption lines is discussed in detail in [16]. Here the mechanical vibrations of the apparatus, and variations in the density and thickness of the air layer might cause each of the "blurs" to shift, but the separation between "blurs" remains unchanged. It is feasible to measure from a single interferogram the changes in the concentrations of two components, for example, when the interfering beam passes through a mixture of metal vapor and inert gas. Of course, it is necessary to know the oscillator force f.

9. Accuracy of Interference Method

The errors that arise in the evaluation of interferograms are about one-tenth of a fringe when evaluating the fringe shift $S(x, y)$, and about 1/500 of a fringe when using the method of superposition. From the viewpoint of measuring the changes in gas density at the low pressures that occur, say, in glow discharges, we may cite the following for illustration: In nitrogen at a pressure of 1 mm Hg, a shift by one-tenth of a fringe corresponds to a change in density $\Delta\rho/\rho \simeq 4$. This applies to light of wavelength 5000 Å, and to a thickness $d = 5$ cm for the region of changed density.

The principal other sources of error are mechanical vibrations
of the equipment, imperfection of the system's optical part, changes
in air density and temperature, etc. [1]. When measuring the electron
concentration and the product Nf, it is necessary to take into account
the effect of the other terms in the dispersion formula (the mutual ef-
fect of the individual lines' anomalous dispersion) [15].

10. Applications of the Interference Method in Plasma Physics

In the field of plasma physics, the number of works employing optical
interferometry is considerable, and their review up to 1964 is presented
in [17]. Here we will concentrate on presenting several typical examples,
to demonstrate the historical development, and also the diversity of the
fields in which the Mach-Zehnder interferometer (respectively the Jamin
interferometer) is used.

Mach-Zehnder interferometers are standard equipment of many labor-
atories that measure the flow of gases [1], the course of the gas temper-
ature over heated surfaces [9], etc. Interferometers of this type have
already been used in the diagnostics of discharges that are accompanied
by changes in gas density and temperature. The effect of the electron
concentration upon the interference pattern was first established in
experimental investigations of the spark-discharge channel's development
over time [18, 19]. Other works, for example [20, 21], measured the
changes in the density of a discharge lamp's filling, due to arcing.
Work [20] measured the change in the cesium vapor's pressure at the
anode and cathode, during arcing. Within the limits of the sensitivity
offered by Rozhdestvenskiy's method of hooks, it was established that
at currents of 0 to 8 A and vapor pressures of about 0.01 to 0.1 mm Hg,
the vapor pressure at the anode did not depend on the passage of current
through the discharge lamp. With a combination of the method of hooks
and of the method of superposition, however, the change measured in the
concentration of cesium atoms was $\Delta N \simeq 10^{13}$ cm^{-3} at a current of 5 A,
which was less than 1 percent of the original value. No change in va-
por pressure was found at the cathode. Work [21] measured the change
in gas density inside the tube during a discharge in argon. The tube
was 1 m long and 6 cm in diameter. The current, in the form of rectan-
gular pulses of about 200 μsec duration, reached 600 A. At a pressure
of 6 mm Hg it was found that in a certain stage of the discharge the
gas density inside the tube was insignificant, not exceeding 10 percent
of the original value.

The feasibility of measuring the concentrations of electrons
by interferometry was again investigated independently, in shock-wave
experiments [22, 23, 24]. To separate the shifts caused by free elec-
trons and by a change in gas density, Alpher and White used measurements
at two wavelengths (the usual method of evaluation requires two fringe
shifts for determining the two quantities), and they called attention to

the increase in sensitivity when using light of a longer wavelength.
They noted also the different direction of the shifting of the individu-
al fringe and the loci of greatest contrast, when white light was used
in the optical interferometry of ionized gases. Even so it is possible
to distinguish on one and the same interferogram the changes in the re-
fractive index due to a change in density, from the changes in the re-
fractive index due to the ionization of the gas. The typical values at
which the measurements were conducted [22] were as follows: initial
argon pressure, 3.1 mm Hg; Mach number of shock wave, 15; ratio of
densities, $\rho/\rho_0 = 7$; degree of ionization, 12 percent; measured elec-
tron concentration in a tube 15 m long of square cross section with
sides 8.2 cm, $8.40 \cdot 10^{16}$ cm^{-3}.

The Mach-Zehnder interferometer is one of the most commonly used
instruments in the diagnostics of shock waves [25, 26, 27]. In 1934,
Ladenburg [28] was the first to measure with an interferometer the con-
centration of excited atoms in a glow discharge. But he himself did not
measure simultaneously the electrical characteristics of the discharge.
Complete measurements were made in 1948-1950, by Yu.M. Kagan and N.P.
Penkin, in a low-pressure glow discharge in mercury [29]. They investi-
gated the occupancy of the individual electron levels. They found sig-
nificant departures from equilibrium occupancy, at low current densities
and low pressures. At higher current densities and pressures, the oc-
cupancy approximates the equilibrium occupancy, but not simultaneously
in all levels. In comparison with the methods employing direct emis-
sion or absorption of light, the method of anomalous dispersion is the
least sensitive for determining Nf, but it is the most ac urate. For
when the product Nf is, measured with the help of light emission or ab-
sorption, there occurs repeated absorption or emission of photons, and
this reduces the accuracy [29, 30].

Measurement of the electron concentration in a glow discharge does
not come into consideration because it is feasible to measure concentra-
tions only when they reach an order of 10^{14} cm^{-3}. Although the use of
significantly longer wavelengths has reduced this value, even a concen-
tration of $6.3 \cdot 10^{12}$ that is cited in work [39] as the lower limit (the
readings being accurate to one-tenth of the wavelength, and using a
CN gas laser of wavelength 337 microns as the source of infrared light)
exceeds by two to three orders of magnitude the concentration of elec-
trons in a glow discharge. However, concentrations of this order have
already become common in experimental installations with hot plasma.
In this field the Mach-Zehnder interferometer was used, for example, in
works [31] and [32], to demonstrate the constriction of the plasma.
The electron concentration measured in [31] near the axis was 10^{17} cm^{-3}.
In combination with a laser beam, an interferometer was used to measure
the plasma during magnetic constriction, after intensive preliminary
heating [33]. Also work [34] reports an apparatus with two ruby lasers
and a Mach-Zehnder interferometer used to measure the development, in
space and time, of the controlling magnetic field, of the electron

concentration, and of the shock-wave velocity; furthermore, the development, in space and time, of the concentration of electrons in the current sheet of a magnetically controlled electrodeless discharge in quiescent cold gas (in argon of pressure 0.5 to 10 mm Hg). To measure the magnetic field, use was made of its effect upon rotating the polarization plane of light through flint glass (Faraday's magneto-optical effect); see also work [35]. The measured electron concentrations were of the order of 10^{15} to 10^{16} cm^{-3}. The degree of ionization was about 90 percent.

Using the Jamin interferometer, an electron concentration of about $7.6 \cdot 10^{14}$ cm^{-3} was measured in high-frequency discharge in argon [36], which corresponded to a change in the refractive index $\Delta n = n - 1 = -1.03 \cdot 10^{-7}$. The gas pressure was 0.5 mm Hg; the tube was 32 cm long and 20 mm in diameter. A high-frequency kilowatt source was used. For the given values of the change in the refractive index, the shift of the interference fringes was 0.03 to 0.06 fringe separation.

The cited examples illustrate the range of optical interferometry's applications. Besides the restrictions already mentioned, we must point out the difficulties of measuring electron concentrations of a higher order than about 10^{18} cm^{-3}. In such cases, the light emitted by the plasma itself causes a very strong disturbance [17].

11. Conclusion

During the past few years, optical interferometry has found extensive application in plasma diagnostics, and the number of works in this field is rising steadily. The discovery of lasers has helped to solve the problem of easily controlled pulsed sources of light, and this has permitted the development of measuring the time dependence of the characteristics. The use of wavelengths from the far infrared further expands the measuring range, in the direction of lower electron concentrations. In the 10^{14}--10^{18} cm^{-3} range, optical interferometry is one of the few methods that give satisfactory results. The interferogram contains a significant concentration of information on the behavior of the measured characteristics in space, respectively in time. Electronic computers speed up and improve the utilization of such data.

The most frequently used apparatus in conventional optical interferometry is the Mach-Zehnder interferometer that has a number of advantages over the other types.

The range and accuracy of the measurements have been increased by using the configuration of an interferometer with a crossed spectrograph (Puccianti's method). D. S. Rozhdestvenskiy developed the method of hooks that is used to measure the product Nf. The combination of Puccianti's method with the method of superposition gives approximately a fiftyfold increase in accuracy.

In the review of the experiments, the ranges are given of the individual characteristics, and the parameters of the medium at which optical interferometry can be used to advantage.

The present progressive methods of optical interferometry include the location of the measured region directly inside the laser cavity. This configuration permits the measurement of concentrations already from about 10^{10} cm^{-3}, and the measurement of electron and ion temperatures [39]. Mention should be made also of the use of interferometers in holgraphy [37], respectively in interference holography that eliminates the need of incoming and outgoing walls of plane and parallel plates, and has already reported the first results [38].

But it seems that not even the conventional methods have reached their maximum application as yet. This applies to measurements with the Mach-Zehnder interferometer, and to the methods of application that were described here. In this context we wish to note that there exists relatively little mutual knowledge about the results of the Western countries and in the Soviet Union, as evident also from the different names of the same interferometer.

The advantages and universality of the methods of optical interferometry predetermine a further expansion of their applications in plasma diagnostics.

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